Generalized algorithm for efficient multi-channel data fusion and real-time implementation using wavelet transform

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Outline

- Motivation
- Problem formulation
- Bayesian Estimation for fusion
- Numerical optimization for real-time implementation

Multispectral remote sensing

- Some recent instruments (satellite based)
  - Landsat mission:
    - **Multi-Spectral Scanner (MSS) – 4 bands**
    - **Thematic Mappers – 7 bands**
  - Terra:
    - **Advanced Spaceborne Thermal Emission and Reflection Radiometer (ASTER) – 14 bands**
    - **Moderate Resolution Imaging Spectrometer (MODIS) – 36 bands**
  - EO-1:
    - **Hyperion – 220 bands**
    - **Advanced Land Imager (ALI) – 10 bands**

Characteristics of sensor measurements

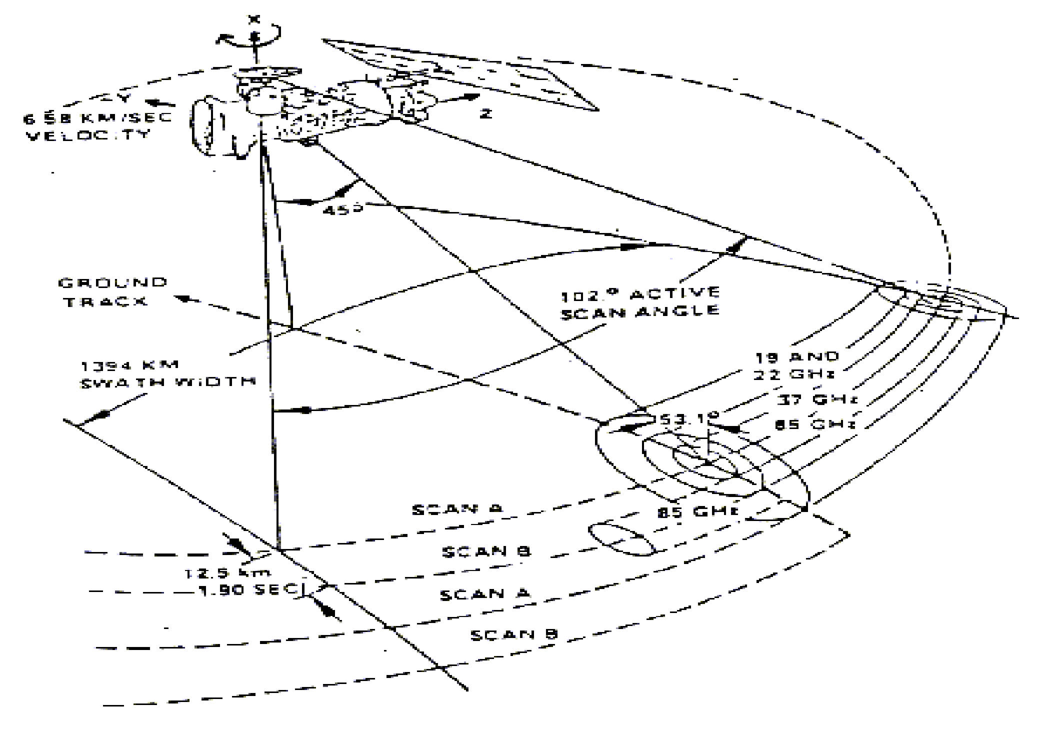
- Sizable acquired data at different resolutions
- Missing/erroneous data
- Non-stationarity
- Multispectral/hyperspectral
- High-resolution (as low as 1m)
- Correlated channels
- Spatial dependencies

Objectives

- **Data exploitation for analysis and interpretation:**
  - **Coherent & composite image by *sensor Fusion***
  - **Enhanced resolution of fused data by *Optimal Estimation***
  - **Parsimonious and flexible representation of non-stationary data by *Statistical Transformations***
- **Processing guidelines:**
  - **Memory efficiency**
  - **Real-time implementation**
  - **FPGA compatible algorithms**
  - **Minimize communication burden**

The SSM/I instrument

- Special Sensor Microwave/Imager



- Problem: Jointly exploit channels for resolution enhancement

Problem Formulation

- **The channel measurements Y are given as:**

$$\mathbf{Y} = \mathbf{G}\mathbf{X} + \mathbf{E}$$

where
G = The antenna gain function
X = The true underlying temperature field
E = Measurement error

- **Assuming Gaussian model for X and E,**
with *Bayesian Estimation* we have:

$$\hat{\mathbf{X}} = (\mathbf{P}^{-1} + \mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{G})^{-1}\mathbf{G}^{T}\mathbf{R}^{-1}\mathbf{Y}$$

P = *a priori* **Covariance Matrix of X**
R = *a priori* **Covariance Matrix of E**

Formulation for SSM/I

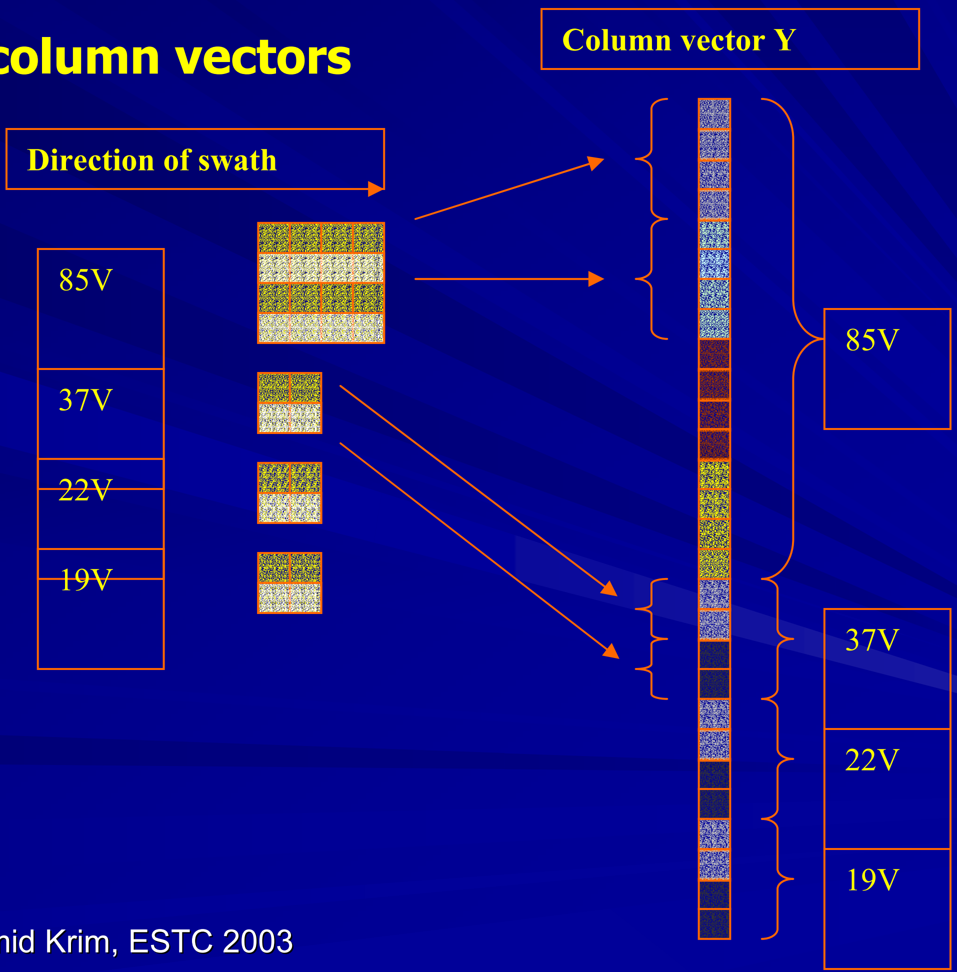
- Format *stationarized* vertical polarization channels at each frequency from the seven available data channels

- Vectorize 2-D data into 1-D column vectors

$$\mathbf{Y}_{85} = \mathrm{Vec}[Y_{85}]$$

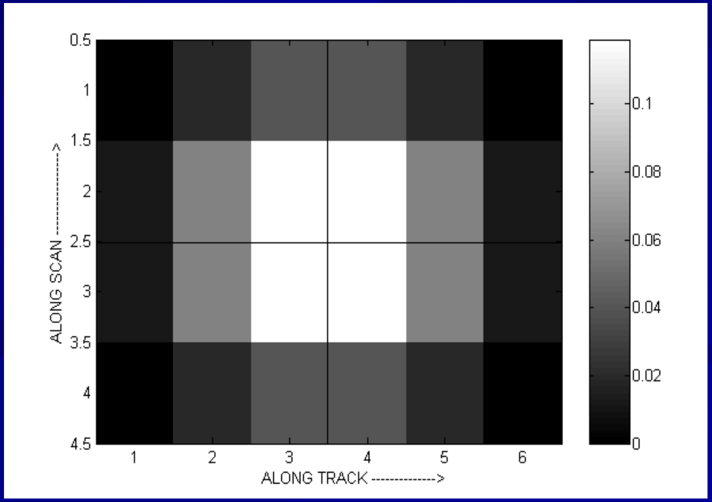
- Append all channels

$$\mathbf{Y} = [\mathbf{Y}_{85}, \mathbf{Y}_{37}, \mathbf{Y}_{22}, \mathbf{Y}_{19}]^T$$



Receive Antenna model

- **Hypotheses**
  - **Local stationarity over 10x10 pixel patches in the 85V channel**
  - **Estimated field finer in resolution 4 times in each dimension than the 85V channel i.e. pixel width = 12.5/4 = 3.125km**
  - **Each of the four channels has a jointly binomial gain pattern**
  - **Example: Gain pattern for the 85V channel (note implied overlap along track)**



Construction of statistical models

- Empirically estimate *a priori* covariance matrices
  - P of the field
    - -Challenge from non-stationarity
  - R of the measurement error
    - -Use as weights on channels

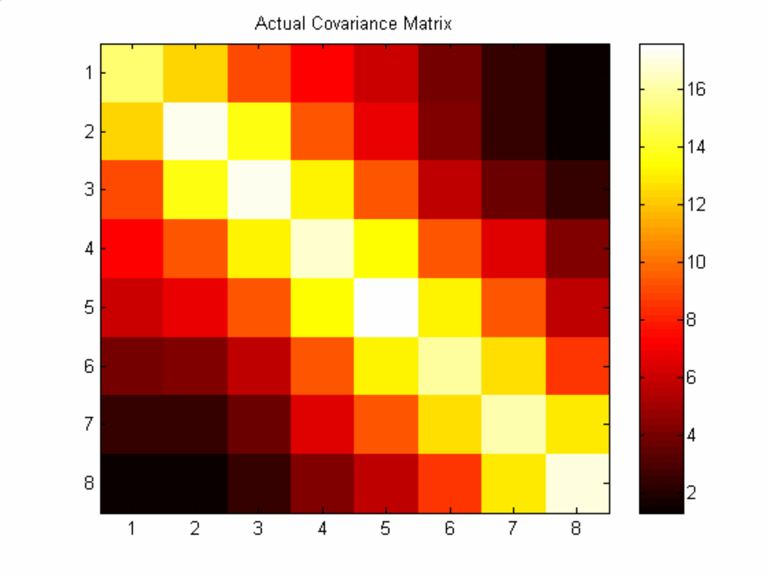
Field prior covariance model

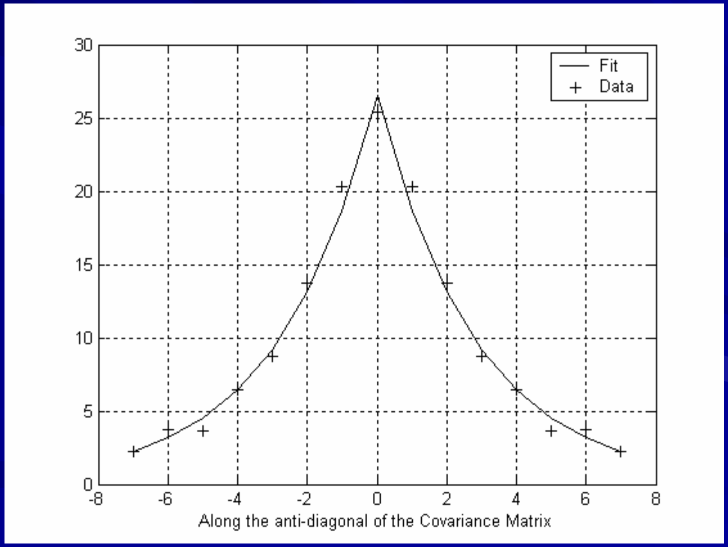
- **Assumptions and method**
  - **Use 85V channel**
    - **-Closest in resolution to underlying field**
    - **-Channel with least overlap of footprints**
    - **-No overlap in scan direction**
  - **Windowing**
    - **-Local stationarity in general**
    - **-Global stationarity achieved for locally detrended data**
    - **-Mean normalization over 8x8 shifting window**

Field prior covariance model

- **Assumptions and method**
- **Compute statistics for along scan direction**
    - **-Fits to exponential model in the anti-diagonal**
    - **-Only two parameters required to define model**

$$C(d) = A * \exp(B\,|d|)$$





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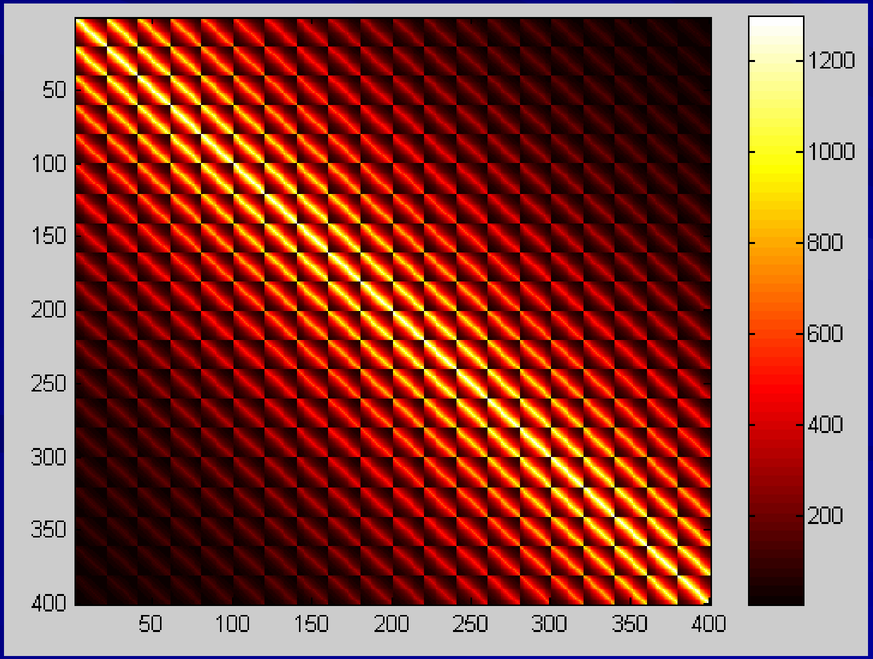
Field prior covariance model

- **Assume isotropy**
- **Mathematical model for vectorized data of NxN**

$$P(x,y) = C(d) =$$

$$= C(\sqrt{(\lfloor x/N \rfloor - \lfloor y/N \rfloor)^2 + (x - \lfloor x/N \rfloor - y + \lfloor y/N \rfloor)^2}$$

$$= A * \exp\{-B * \sqrt{(\lfloor x/N \rfloor - \lfloor y/N \rfloor)^2 + (x - \lfloor x/N \rfloor - y + \lfloor y/N \rfloor)^2}\}$$



More Generally…

- Observe characteristics of sample data to determine what input channel(s) provide statistical data that is closest to the underlying field and thus has minimum overlap
- Apply statistical normalization (e.g. detrend) to selected data to guarantee the imposed assumptions of stationarity
- Compute the covariance matrix for the relevant input data points
- Construct a field covariance model
- Using model, solve for field estimate of a specified swath length.

Error prior covariance model

- Assumptions
  - Zero mean additive white gaussian noise
  - Non-correlation and equal variance for given channel
  - For each channel covariance matrix is diag. $\sigma^2 \mathbf{I}$
  - Interpret the error variance as a weighting factor
  - Higher error variance for a particular channel implies less reliance on the channel in estimating underlying field
  - Impose $\sigma_4{}^2 > \sigma_3{}^2 > \sigma_2{}^2 > \sigma_1{}^2$

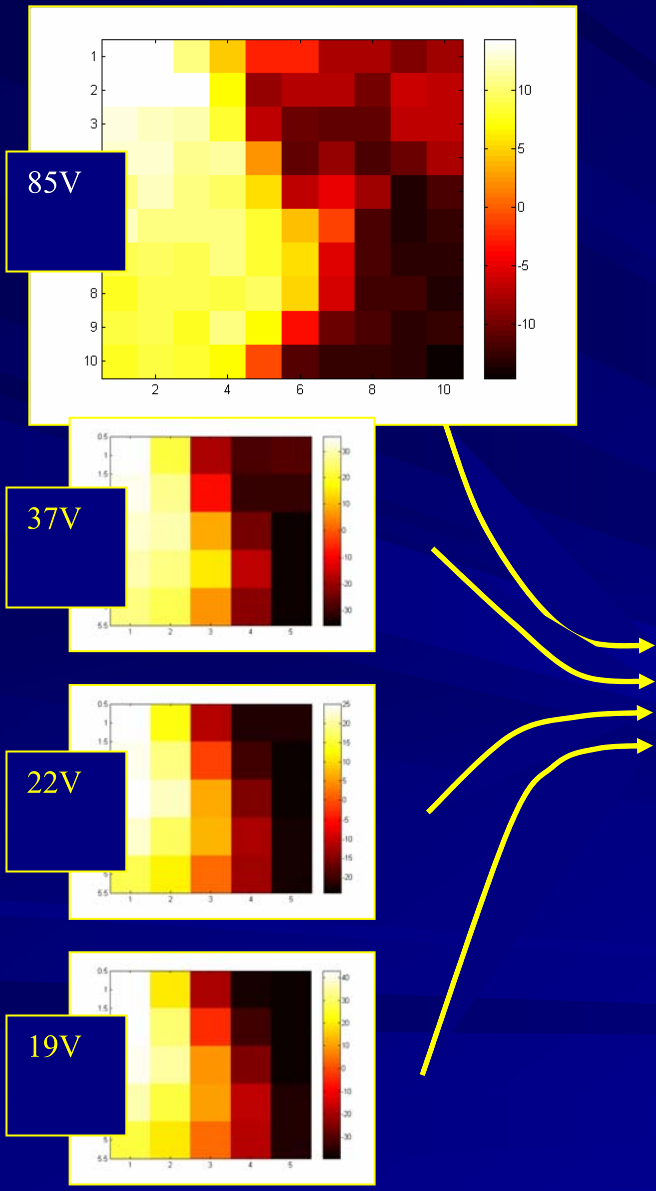
Prior of measurement error

- **Final Covariance Matrix**

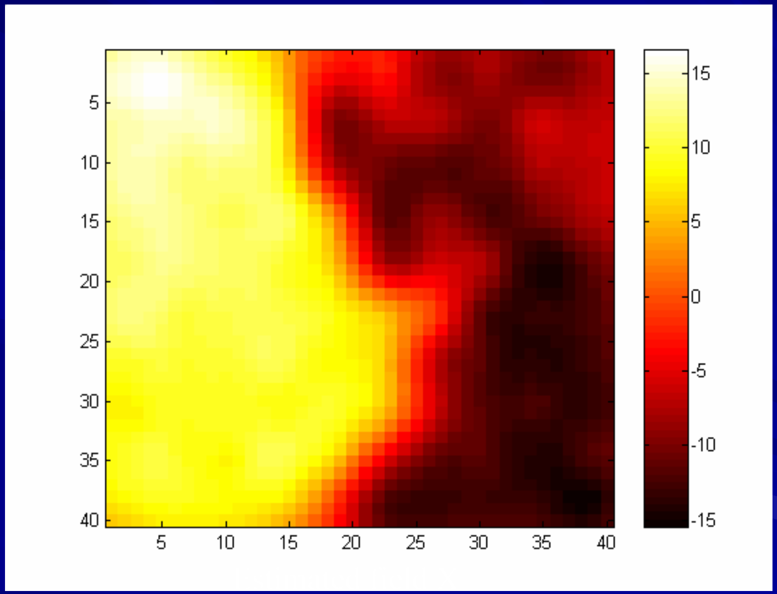
$$\mathbf{R} = \begin{bmatrix} \sigma_1^{\,2} I_{n^2 x n^2} & 0 & 0 & 0 \\ 0 & \sigma_2^{\,2} I_{n^2/4 x n^2/4} & 0 & 0 \\ 0 & 0 & \sigma_3^{\,2} I_{n^2/4 x n^2/4} & 0 \\ 0 & 0 & 0 & \sigma_4^{\,2} I_{n^2/4 x n^2/4} \end{bmatrix}$$

- **Zeros indicate assumption of independence of measurement errors of various channels**
- **Note size dependency on field size for different channels**

Direct method experiment



$$\hat{X} = (P^{-1} + G^T R^{-1} G)^{-1} G^T R^{-1} Y$$



Pre-whitening manipulation…

- **Take Cholesky Factorization of Field priori covariance matrix**

$$\mathbf{P} = \mathbf{A}\mathbf{A}^{T}$$

- **A is a full rank Upper Triangular matrix**
- **Let** $\mathbf{F}_{W} = \mathbf{A}^{-1}$
- **Rewrite problem statement**

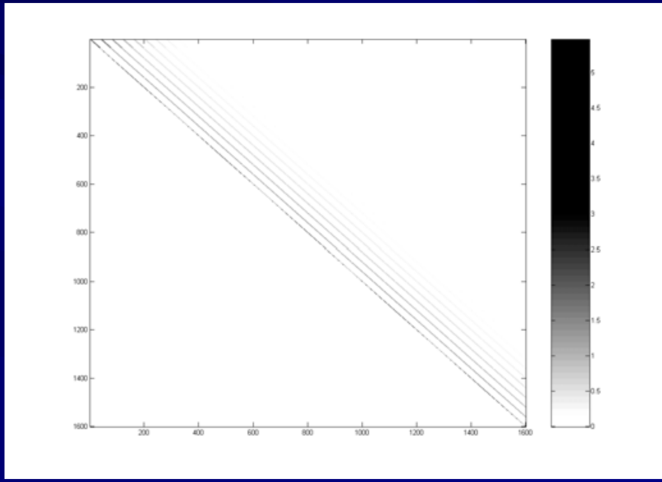
$$\mathbf{Y} = \mathbf{G}\mathbf{F}_{W}^{-1}\mathbf{F}_{W}\mathbf{X} + \mathbf{E} \qquad \mathbf{G}_{W} = \mathbf{G}\mathbf{F}_{W}^{-1}$$

- **The estimation solution now is:**

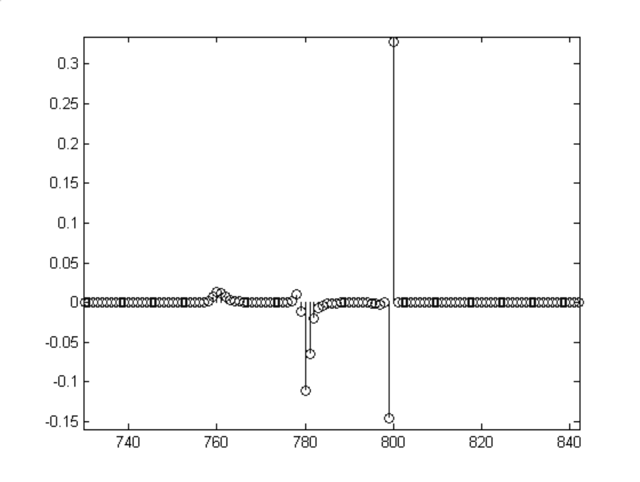
$$\hat{\mathbf{X}}_{W} = \mathbf{F}_{W}\hat{\mathbf{X}} = (\mathbf{I} + \mathbf{G}_{W}^{T}\mathbf{R}^{-1}\mathbf{G}_{W})^{-1}\mathbf{G}_{W}^{T}\mathbf{R}^{-1}\mathbf{Y}$$

Pre-whitening transform

- The matrix $\mathbf{F}_W$ is in effect a whitening filter.



Inverse filter matrix

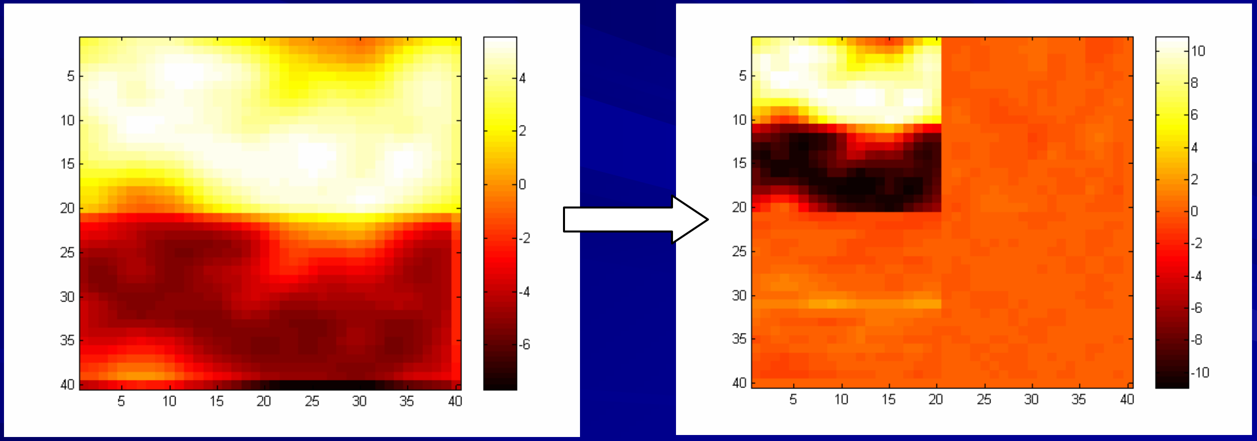


Impulse response of whitening filter

- We use $\mathbf{F}_W^{-1}$ i.e. $\mathbf{A}$ to "recolor" the estimated quantity as the final step in estimation process

Wavelet/Sparse preconditioning

- Take advantage of sparseness resulting from wavelet transform
- Simplified choice of a suitable wavelet with pre-whitening in place
- Level-1 wavelet decomposition of the estimated quantity (reformatted as 2-D image) using a 2-D Haar wavelet.



Wavelet preconditioning

- Thus, application of a wavelet transform to the vectorized white data
  - Threshold and preserve significant portion of information
  - Data size reduction by a factor of 4
- Defining

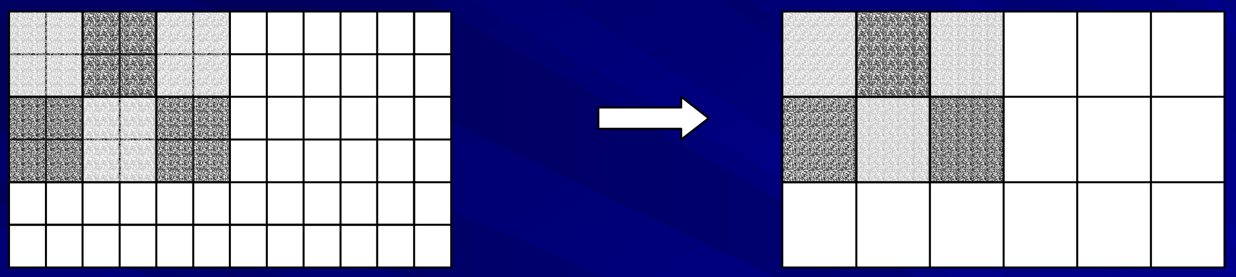
$$X_l = W_l X_W \qquad\qquad G_l = G_W W_l^T$$

- We have

$$\hat{X}_l = W_l F_W \hat{X} = (I + G_l^T R^{-1} G_l)^{-1} G_l^T R^{-1} Y$$

Effect of preconditioning

- **Haar wavelet transform**



$\hat{\mathbf{X}}_W$ $\qquad$ $\mathbf{W}_1\hat{\mathbf{X}}_W$

- **Size reduction of matrices $\mathbf{X}_1$ and $\mathbf{G}_1$ by a factor of 4 in both dimensions**
- **Significant decrease in computational cost in inverting**

$$(\mathbf{I}+\mathbf{G}_1{}^T\mathbf{R}^{-1}\mathbf{G}_1)^{-1}$$

- **Compact representation reduces communication burden for satellites with a small added overhead from preconditioning transforms**

Real-time implementation

- **Estimation based on**
  - **Decorrelating input channels**
  - **Combination of decorrelated channels**
  - **Reconditioning of combined preconditioned estimation**

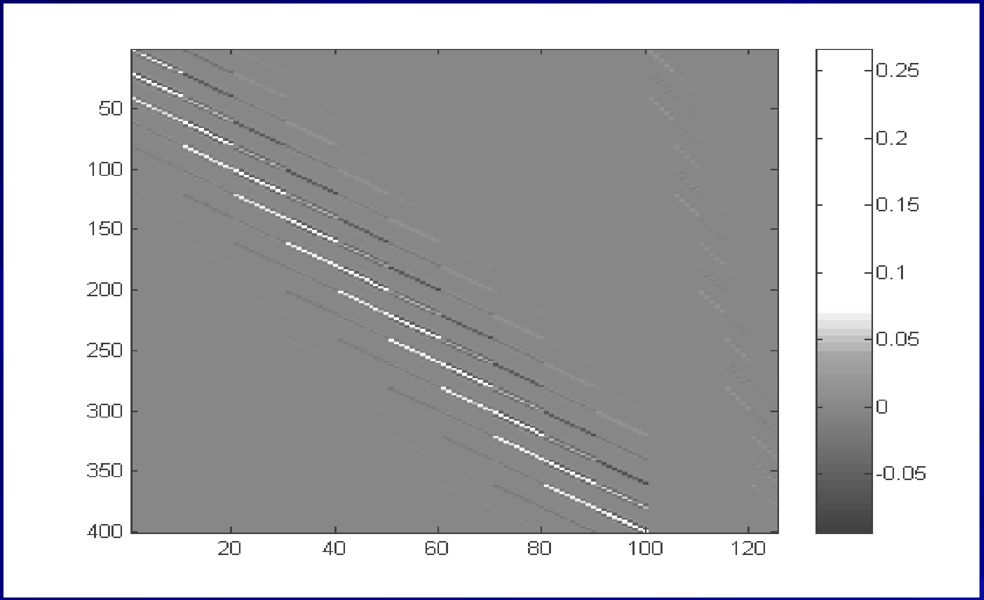
$$\hat{X}_1 = W_1 F_W \hat{X} = (I + G_1{}^T R^{-1} G_1)^{-1} G_1{}^T R^{-1} Y$$

- **Consider,**

$$M = (I + G_1{}^T R^{-1} G_1)^{-1} G_1{}^T R^{-1}$$

Real-time implementation

- Note: only 85V and 37V used for this portion for simplified analysis



$$\mathbf{M} = (\mathbf{I} + \mathbf{G}_1{}^T\mathbf{R}^{-1}\mathbf{G}_1)^{-1}\mathbf{G}_1{}^T\mathbf{R}^{-1}$$

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Real-time implementation

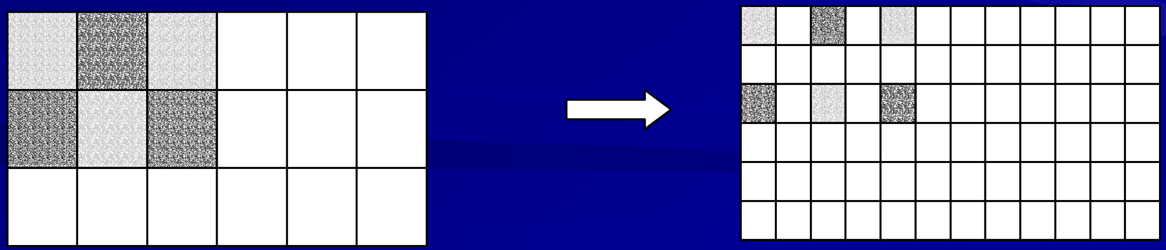
- **Observations:**

$$\mathbf{M} = [\mathbf{M}_{85} \quad \mathbf{M}_{37} \quad .....]$$

$$\hat{\mathbf{X}}_1 = \mathbf{MY}$$

$$= [\mathbf{M}_{85} \quad \mathbf{M}_{37} \quad \ldots] . \begin{bmatrix} \mathbf{Y}_{85} \\ \mathbf{Y}_{37} \\ \ldots \end{bmatrix} = \mathbf{M}_{85}\mathbf{Y}_{85} + \mathbf{M}_{37}\mathbf{Y}_{37} + \ldots$$

- **Input regridding by insertion of zero points**



Real-time implementation

- Modified input:

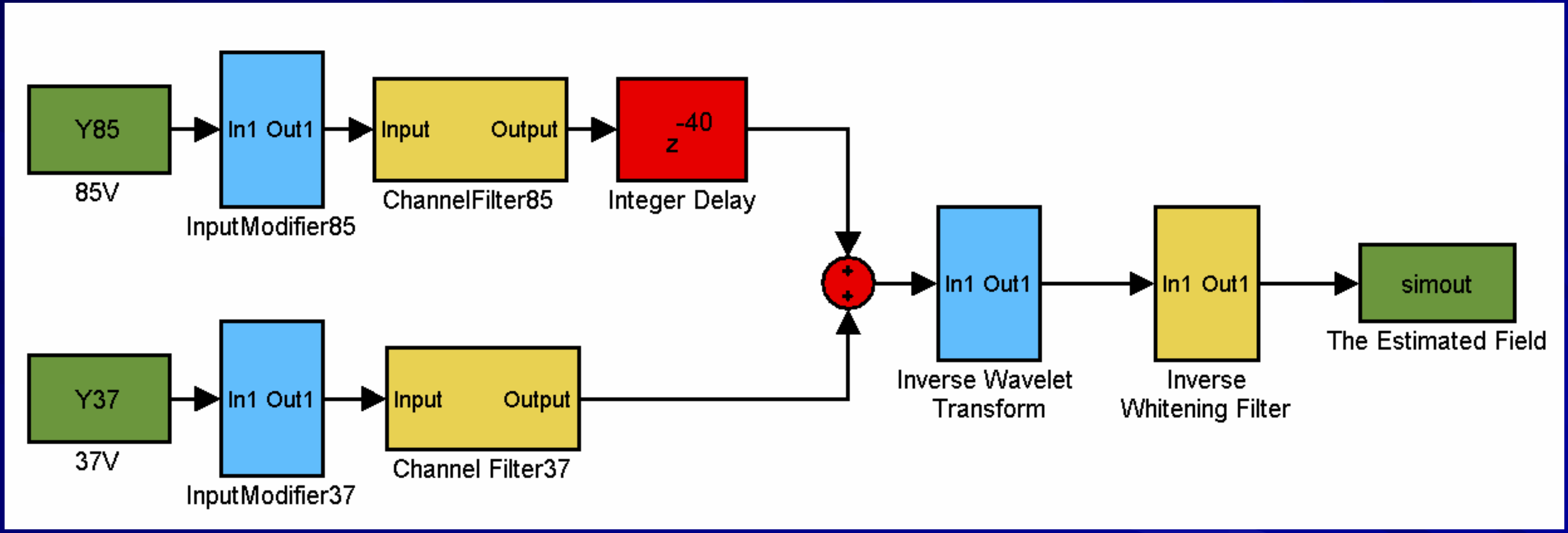
$$Z_{85}, Z_{37}, \ldots$$

- Modified operators are channel filters
- Resulting estimation process:

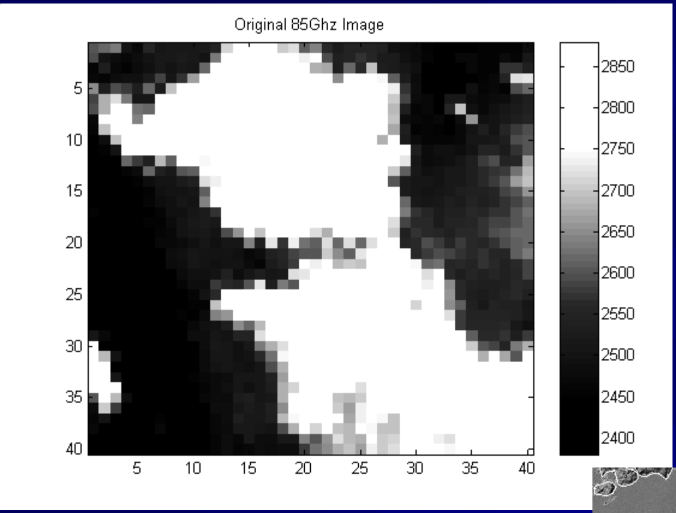
$$\therefore X_1 = F_{85} Z_{85} + F_{37} Z_{37} + \ldots.$$

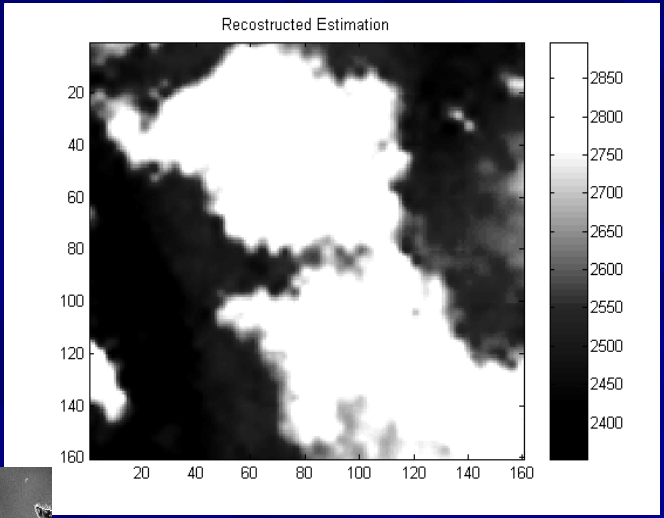
$$\therefore X_1 = f_{85} * Z_{85} + f_{37} * Z_{37} + \ldots.$$

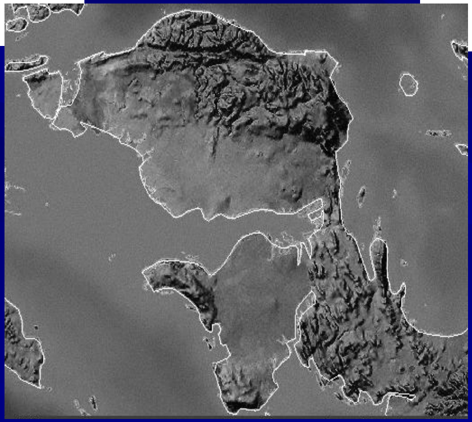
Block diagram



Simulation result







Comparison of original input 85GHz data with reconstructed estimation fusion result

Conclusions

- ✓ An optimal Bayesian estimator for data/sensor fusion developed
- ✓ Empirical analytical models may be constructed and utilized in improving computing efficiency
- ✓ Further improvement and potential near- or real time implementation
- ✓ Simplified and adapted algorithmic architecture for Hardware transitioning

Future Work

- Efficient adaptation to non-stationarity
  - Real-time empirical estimation of covariance model parameters
  - Input data from 85V channel may be used on account of least overlap direction and isotropy
  - Prediction based on feedback from estimated underlying field
- Adapting to instrumental errors
  - Adaptively control the variance of error measurement for every channel in the error covariance matrix
- Investigate other data (e.g. Hyperion data)

Thank You!